

# Bellman Equations in Optimal Stochastic Control and Beyond

Teemu Pennanen<sup>1</sup> and Ari-Pekka Perkkiö<sup>2</sup>

<sup>1</sup>King's College London, UK

<sup>2</sup>Ludwig-Maximilian University of Munich, Germany

## Abstract

We study generalized forms of stochastic control problems and associated Bellman equations whose solutions (Bellman functions) provide computationally attractive optimality conditions for the control problem. Our focus is on problem structures that allow for Bellman functions on low-dimensional spaces. The dimensionality is critical for the complexity of computational techniques, including those based on state space discretization or cutting planes, such as the Stochastic Dual Dynamic Programming (SDDP) algorithm. Existence of solutions to the Bellman equations is established under general conditions that do not require compactness. In the convex case, the existence conditions can be seen as far-reaching generalizations of the no-arbitrage conditions in mathematical finance. We allow for general randomness but show that, in the Markovian case, the dimensionality of the Bellman equations reduces with respect to randomness just like in more traditional control formulations. Besides the theoretical results, we propose a version of the SDDP algorithm applicable to the extended control format. If time permits, we will also discuss applications to intraday electricity trading. Most of the presented material is based on the recent articles [1] and [3] and the book [2].

## References

- [1] J.-P. Chancelier, M. De Lara, T. Lindgaard, F. Pacaud, T. Pennanen, and A.-P. Perkkiö. Optimal operation and valuation of electricity storage systems in intraday markets. *Computational Management Science*, 2025.
- [2] T. Pennanen and A.-P. Perkkiö. *Convex Stochastic Optimization*. Springer Cham, 2024.
- [3] T. Pennanen and A.-P. Perkkiö. Dynamic programming and dimensionality in convex stochastic optimization and control. *Optimization Letters*, 2025.